PHYS-434 Physics of photonic semiconductor devices

Series 9 - Basic notions on light emitting diodes

Exercise I : Spontaneous emission rate

1. The maximum of intensity is emitted at the energy where the derivative of $R_{spont}(h\nu)$ is equal to zero.

$$\Rightarrow \left.\frac{d R_{spont}(h\nu)}{d(h\nu)}\right|_{h\nu = h\nu_{peak}} = 0$$

$$\Rightarrow \frac{1}{2}(h\nu - E_g)^{-\frac{1}{2}} \cancel{\exp\left(-\frac{h\nu - E_g}{k_B T}\right)} + (h\nu - E_g)^{\frac{1}{2}} \times \frac{-1}{k_B T} \times \cancel{\exp\left(-\frac{h\nu - E_g}{k_B T}\right)} = 0$$

$$\Rightarrow \frac{1}{2}(h\nu - E_g)^{-\frac{1}{2}} - \frac{1}{k_B T}(h\nu - E_g)^{\frac{1}{2}} = 0$$

$$\Rightarrow h\nu_{peak} = E_g + \frac{k_B T}{2}$$

2. $$R_{spont}^{max} = R_{spont}(h\nu_{peak}) = K_{spont}\left(\frac{k_B T}{2}\right)^{\frac{1}{2}} \cdot \exp\left(-\frac{1}{2}\right)$$

We wish to determine $h\nu = x$ such that: $R_{spont}(x) = R_{spont}^{max}/2$

$$\Rightarrow (x - E_g)^{\frac{1}{2}} \exp\left(-\frac{x - E_g}{k_B T}\right) = \frac{1}{2}\left(\frac{k_B T}{2}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\right)$$

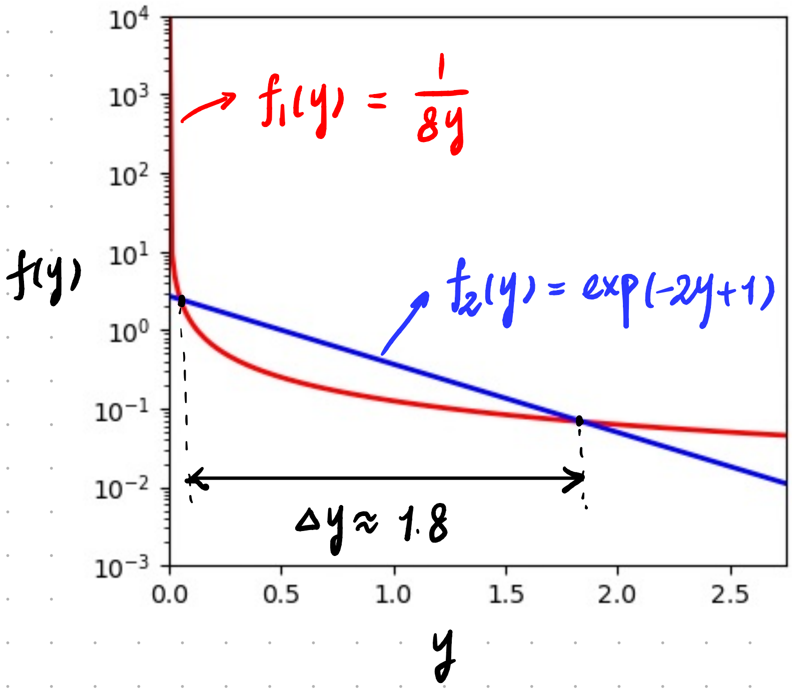
$$\Rightarrow (x - E_g) \exp\left(-\frac{2(x - E_g)}{k_B T}\right) = \frac{k_B T}{8} \exp(-1)$$

$$\Rightarrow \frac{8(x - E_g)}{k_B T} \exp\left(-\frac{2(x - E_g)}{k_B T} + 1\right) = 1$$

* change of variable:

$$y = \frac{x - E_g}{k_B T}$$

$$\overset{*}{\Rightarrow} 8y \exp(-2y + 1) - 1 = 0$$



Graphic solution:

$$\underbrace{\frac{1}{8y}}_{f_1(y)} = \underbrace{\exp(-2y+1)}_{f_2(y)}$$

$\Rightarrow \Delta y \approx 1.8 \quad \Rightarrow \Delta x \approx 1.8\, k_B T$

$\Rightarrow$ FWHM of $R_{spont}(h\nu)$:

$$\Delta h\nu \approx 1.8\, k_B T$$

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3. $\lambda_{peak} = \dfrac{hc}{h\nu_{peak}} \overset{\text{differentiation}}{\Longrightarrow} \Delta\lambda = \dfrac{hc\Delta\nu}{h(\nu_{peak})^2} = \underbrace{\dfrac{hc}{h\nu_{peak}}}_{\lambda_{peak}} \cdot \dfrac{\Delta\nu}{\nu_{peak}}$

$\Rightarrow \Delta\lambda = \lambda_{peak} \cdot \dfrac{\Delta\nu}{\nu_{peak}}$ and $\Delta h\nu \sim 1.8\, k_B T$, $\nu_{peak} = c/\lambda_{peak}$

$\Rightarrow \Delta\lambda = \dfrac{1.8\, k_B T}{hc}\lambda_{peak}^2$

Exercise II: Extraction efficiency and issues related to light extraction

1. Along the normal to the sample, we can write the Fresnel equation as:

$$R(\theta_i = \theta_t = 0) = \frac{(n_{sc}-1)^2}{(n_{sc}+1)^2}$$

η_{diel} corresponds to the transmitted fraction of emitted light:

$$\eta_{diel} = 1 - R \Rightarrow \eta_{diel} = \frac{4n_{sc}}{(n_{sc}+1)^2}$$

and for GaAs: $n_{sc} = 3.6$ $\quad \eta_{diel} \approx 0.68$

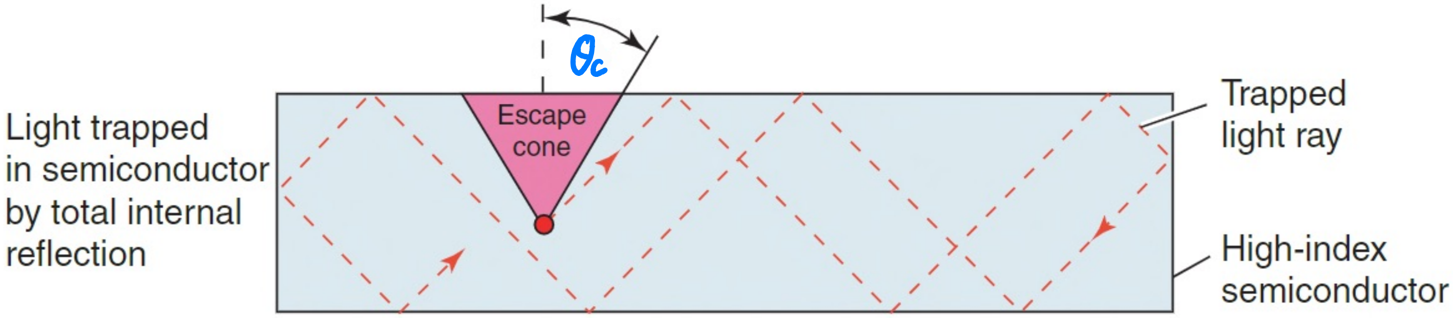
for GaN: $n_{sc} = 2.5$ $\quad \eta_{diel} \approx 0.82$

It is therefore seen that the transmission gets larger when the optical refractive index is smaller, i.e. for LEDs which emit toward the shorter wavelengths (as $n_{sc} \downarrow$ when $E_g \uparrow$)

2. We wish to determine the total fraction of light emitted by a point source (here a quantum well or a quantum dot) which will effectively leave the semiconductor/dielectric medium.

The total solid angle is such that: $\Omega_{tot} = 4\pi$ (sr.)

As only the fraction of light emitted within the light cone defined by the internal critical angle θ_c will effectively leave the dielectric medium,



We get: ① $\theta_c = \sin^{-1}\left(\frac{n_{ext}}{n_{sc}}\right)$ [Snell's law] (in air, $n_{ext} = 1$)

② Internal solid angle: $\Omega_c = 2\pi(1 - \cos\theta_c)$

Reminder: $d\Omega = \sin\theta \, d\theta \, d\varphi$

$$\Rightarrow \Omega_c = \int_0^{2\pi} d\varphi \int_0^{\theta_c} \sin\theta \, d\theta = 2\pi(1 - \cos\theta_c)$$

$$\Rightarrow \Omega_c = 2\pi\left(1-(1-\sin^2\theta_c)^{\frac{1}{2}}\right) = 2\pi\left(1-\left(1-\left(\frac{n_{ext}}{n_{sc}}\right)^2\right)^{\frac{1}{2}}\right)$$

$$= 2\pi \, \frac{n_{sc} - (n_{sc}^2 - n_{ext}^2)^{\frac{1}{2}}}{n_{sc}}$$

The emitted fraction $= \dfrac{\Omega_c}{\Omega_{tot}}$ (considering the extraction by only 1 side)

$$\Rightarrow \frac{\Omega_c}{\Omega_{tot}} = \frac{n_{sc} - (n_{sc}^2 - n_{ext}^2)^{\frac{1}{2}}}{2n_{sc}} \quad \text{for} \begin{cases} \text{GaAs/air}: & \Omega_c/\Omega_{tot} \sim 2\% \\ \text{GaN/air}: & \Omega_c/\Omega_{tot} \sim 4\% \end{cases}$$

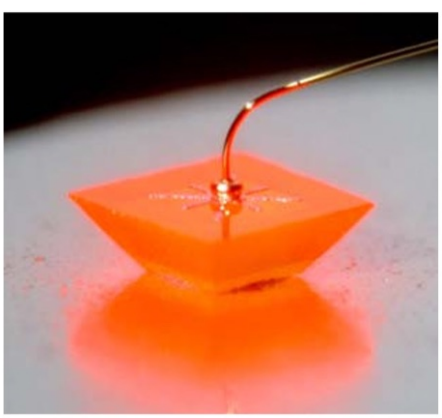
However, LEDs exhibit a larger extraction efficiency thanks to various points:

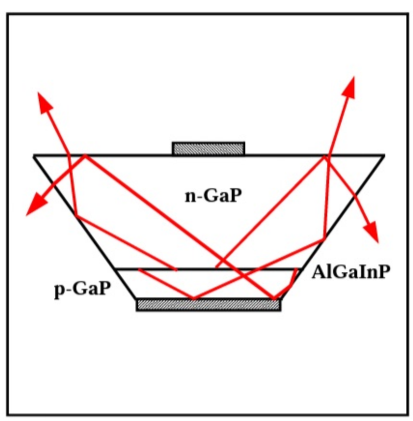
- "photon recycling":

  photons reflected at the semiconductor/air interface are reabsorbed by the medium, creating novel electron-hole pairs. A fraction of them will radiatively recombine and so on and so forth.

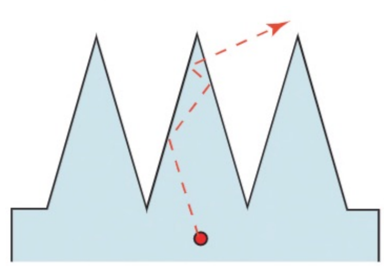
- geometries differ from a simple planar geometry:

  Lumileds AlGaInP/GaP Truncated-Inverted-Pyramid LED (cf. lecture)



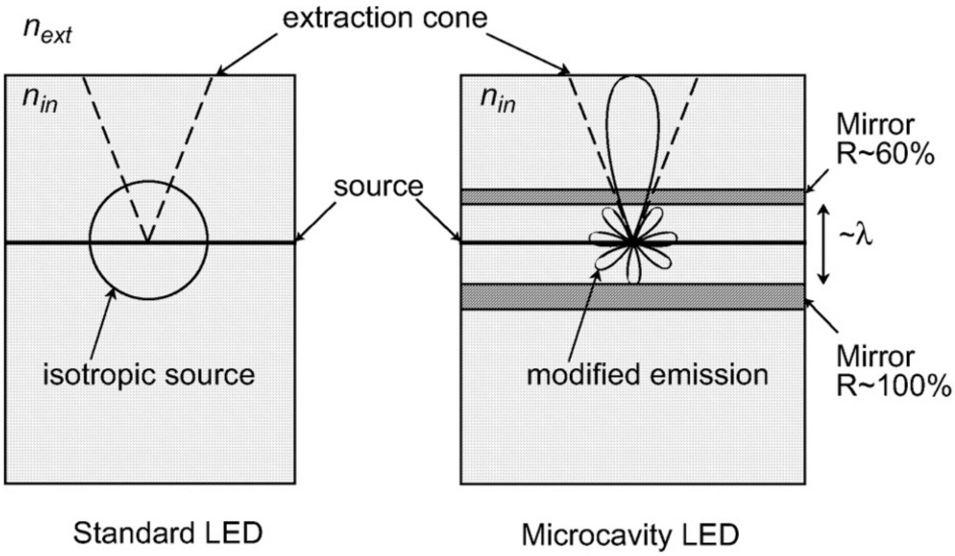


- roughened semiconductor/air interface:



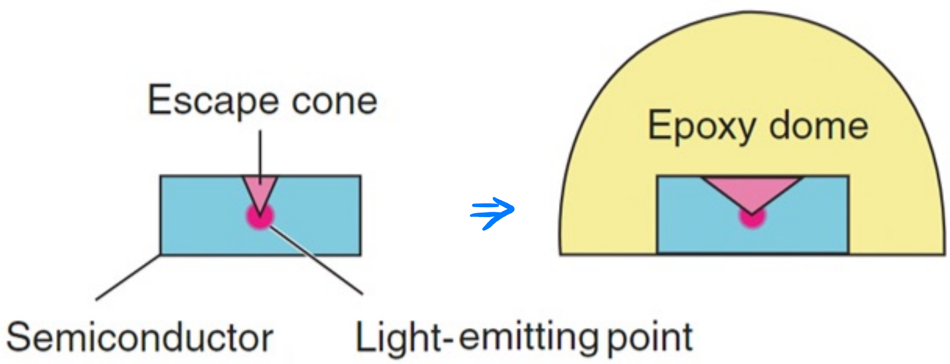
Surface-textured LED

- microcavity effect (RCLED):



the emission is modified by optical confinement
⇒ a large fraction of light is emitted into a resonant mode which is almost entirely inside the light cone.

- use of an intermediate medium ($n_{ext} > 1$):



$n_{ep} \sim 1.5 - 1.57$